Topic 3

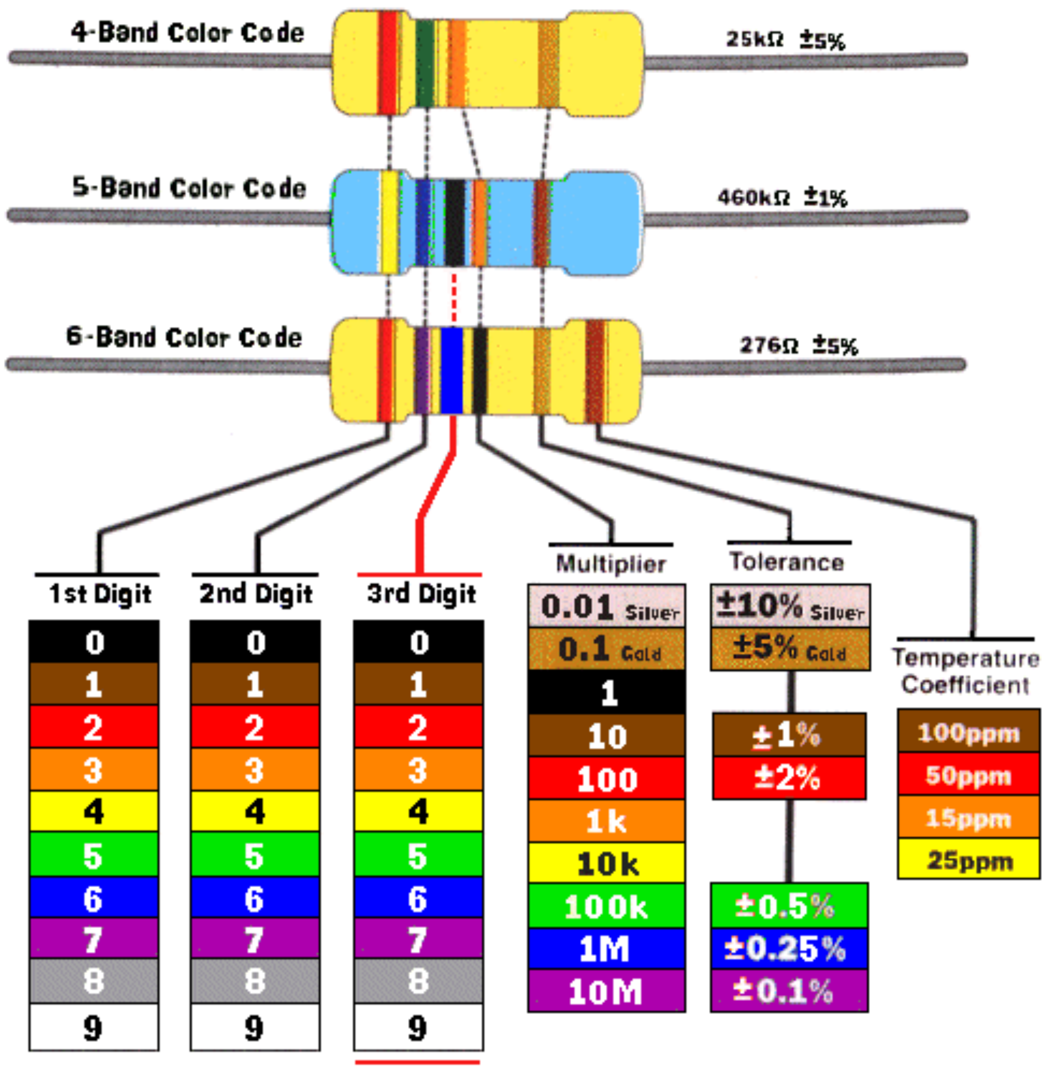
Resistors & Resistor Circuits

Prof Peter Y K Cheung
Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE1_EE/
E-mail: p.cheung@imperial.ac.uk

Resistor parameters and identification

- Resistors are usually colour coded with their values and other characteristics as shown here.
- They also come in different tolerances (e.g. ±0.1% to ±10%).
- Other important parameters are:
  - Power rating (in Watts)
  - Temperature coefficient in parts per million (ppm) per degree C
  - Stability over time (also in ppm)
  - Inductance (don't worry about this for now)
- Resistors can be made of different materials: carbon composite (most common), enamel, ceramic etc.



Resistor – Preferred values

- In theory, resistor values is a continuous quantity with infinite different values.
- In reality, resistor as a component exists within some tolerance (say, ±5% is common)
- Therefore there is NO reason to provide more than selected number of different resistor values for a given tolerance.
- The standard "preferred values" for resistors are given in this table for ±5% (most common), ±10% and ±20%, respectively designated as the E24, E12, E6 series.
- For example, if you need a 31.3kΩ resistor with tolerance of ±10%, you could use a 30kΩ E24 resistor (±5%) instead and still stay within the allowable tolerance.
- Therefore, when computing solutions resistor values for electronic circuits, it is silly to use precision with many digits.

Resistor Values

E6 (20%)	E12 (10%)	E24 (5%)
10	10	10
		11
	12	12
		13
15	15	15
		16
	18	18
		20
22	22	22
		24
	27	27
		30
33	33	33
		36
	39	39
		43
47	47	47
		51
	56	56
		62
68	68	68
		75
	82	82
		91

Units and Multipliers

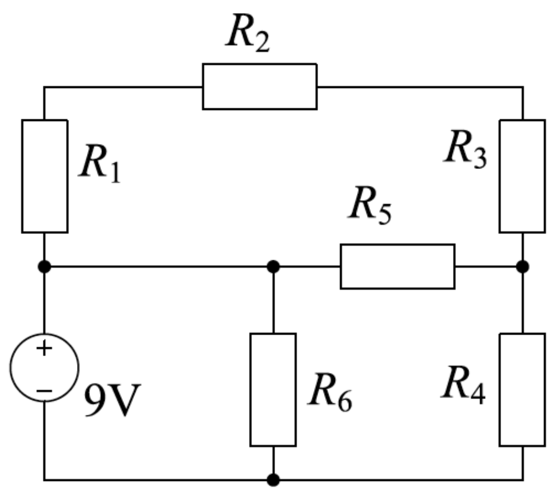
Quantity	Letter	Unit	Symbol
Charge	Q	Coulomb	C
Conductance	G	Siemens	S
Current	I	Amp	A
Energy	W	Joule	J
Potential	V	Volt	V
Power	P	Watt	W
Resistance	R	Ohm	Ω

Value	Prefix	Symbol
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Value	Prefix	Symbol
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Series and Parallel

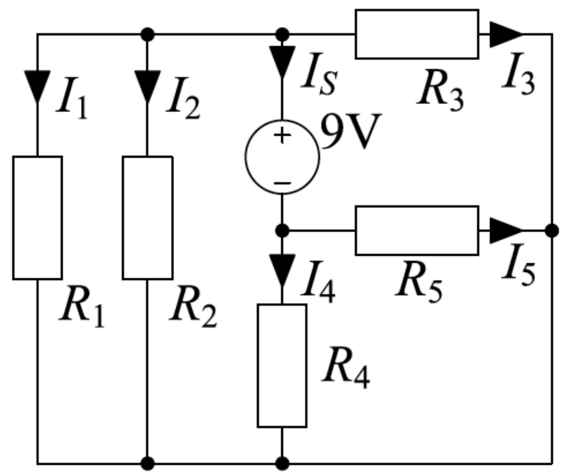
Series: Components that are connected in a chain so that the same current flows through each one are said to be *in series*.



- R_1, R_2, R_3 are in series and the same current always flows through each.
- Within the chain, each internal node connects to only two branches.
- R_3 and R_4 are not in series and do not necessarily have the same current.

Parallel: Components that are connected to the same pair of nodes are said to be in parallel .

- R_1, R_2, R_3 are in parallel and the same voltage is across each resistor (even though R_3 is not close to the others).
- R_4 and R_5 are also in parallel.

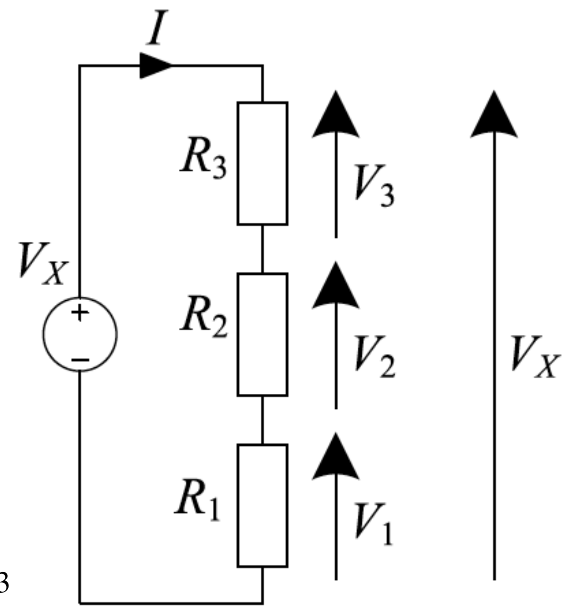


P52-53

Series Resistors: Voltage Divider

$$V_x = V_1 + V_2 + V_3$$
$$= I R_1 + I R_2 + I R_3$$
$$= I(R_1 + R_2 + R_3)$$

$$\frac{V_1}{V_x} = \frac{I R_1}{I(R_1 + R_2 + R_3)}$$
$$= \frac{R_1}{R_1 + R_2 + R_3} = \frac{R_1}{R_T}$$



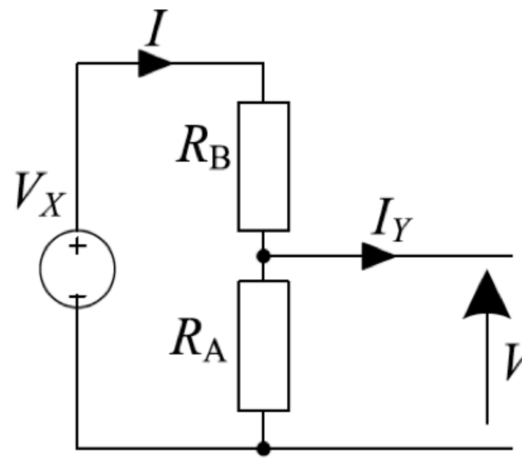
Where R_T is the total resistance of the chain $R_T = R_1 + R_2 + R_3$

V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$

Approximate Voltage Divider:

If $I_Y = 0$, then $V_Y = \frac{R_A}{R_A + R_B} V_X$.

If $I_Y \ll I$, then $V_Y \approx \frac{R_A}{R_A + R_B} V_X$.

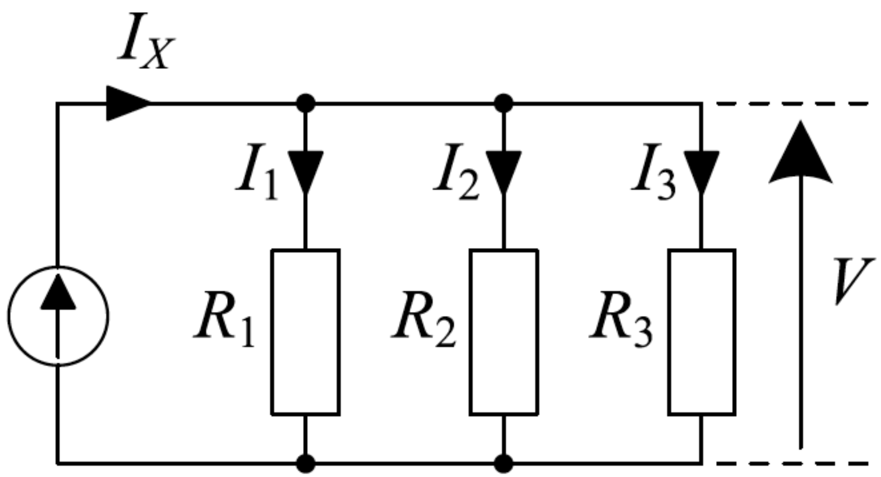


P56-57

Parallel Resistors: Current Divider

- Parallel resistors all share the same V.

$$I_1 = \frac{V}{R_1} = V\ G_1$$ where $G_1 = \frac{1}{R_1}$ is the *conductance* of R_1.



$$\begin{aligned} I_x &= I_1 + I_2 + I_3 \\ &= VG_1 + VG_2 + VG_3 \\ &= V(G_1 + G_2 + G_3) \end{aligned}$$

$$\frac{I_1}{I_X} = \frac{VG_1}{V(G_1+G_2+G_3)} = \frac{G_1}{G_1+G_2+G_3} = \frac{G_1}{G_P}$$

where $G_P = G_1 + G_2 + G_3$ is the total conductance of the parallel resistors.

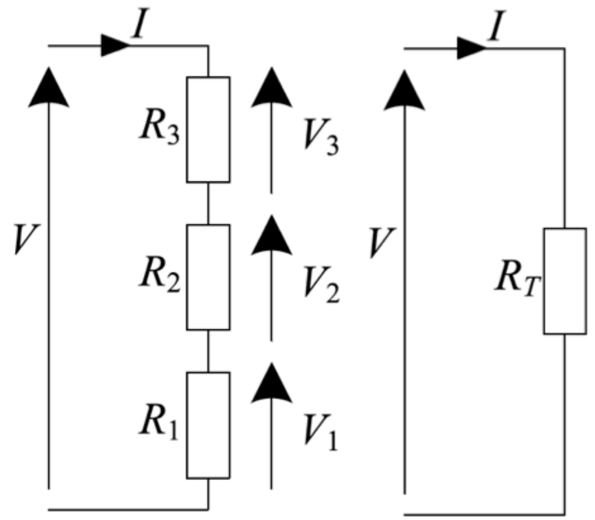
I_X is divided into $I_1 : I_2 : I_3$ in the proportions $G_1 : G_2 : G_3$.

Equivalent Resistance: Series

- We know that

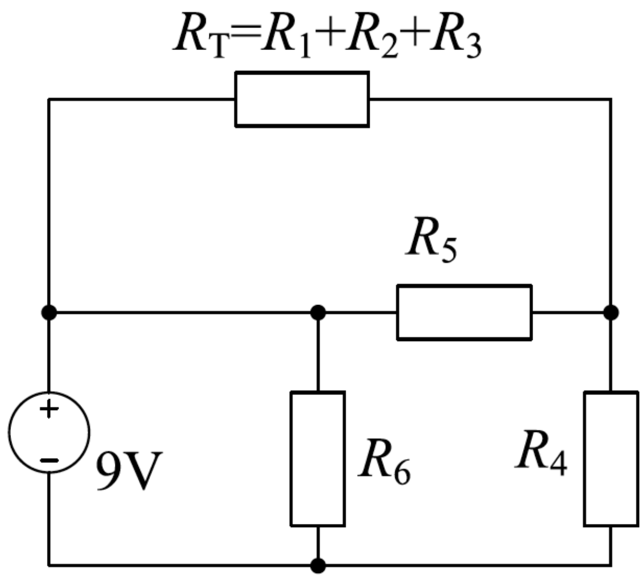
$$V = V_1 + V_2 + V_3 = I\ (R_1 + R_2 + R_3) = I\ R_T$$

- So we can replace the three resistors by a single *equivalent resistor* of value R_T without affecting the relationship between V and I.



- Replacing series resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.

- However the individual voltages V_1, V_2 and V_3 are no longer accessible.

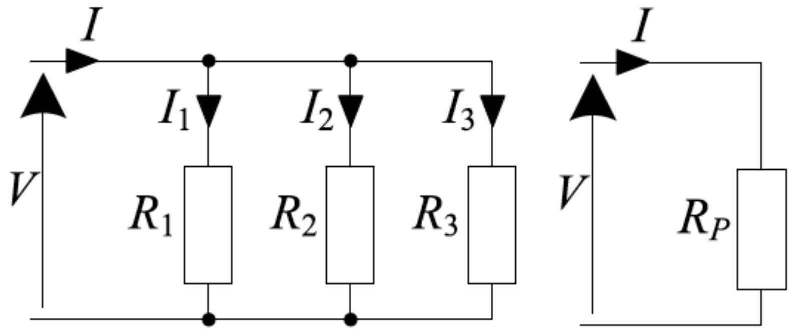


Equivalent Resistance: Parallel

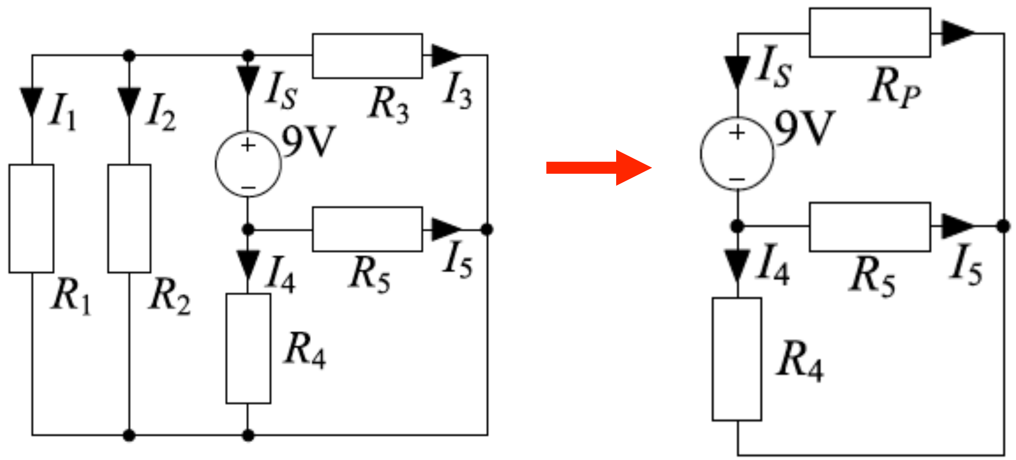
- Similarly we known that

$$I = I_1 + I_2 + I_3 = V\ (G_1 + G_2 + G_3) = V\ G_P$$

- So $V = I\ R_P$ where $R_P = \frac{1}{G_F} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$
- We can use a single *equivalent resistor* of resistance R_P without affecting the relationship between V and I.



- Replacing parallel resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.
- R_4 and R_5 are also in parallel.
- Much simpler - although none of the original currents $I_1, \bullet\bullet\bullet\bullet, I_3$ are now implicitly specified.



Equivalent Resistance: Parallel Formulae

- For parallel resistors $G_P = G_1 + G_2 + G_3$

  or equivalently $R_P = R1 \,||\, R2 \,||\, R3 = \dfrac{1}{1/R_1 + 1/R_2 + 1/R_3}$
- These formulae work for any number of resistors.
- For the special case of two parallel resistors

  $$R_P = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2}$$ ("product over sum")
- If one resistor is a multiple of the other

  Suppose $R_2 = kR_1$, then

  $$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{kR_1^2}{(k+1)R_1} = \frac{k}{k+1} R_1 = (1 - \frac{1}{k+1})\ R_1$$
- Example: $1\ k\Omega \,||\, 99\ k\Omega = \dfrac{99}{100} k\Omega = (1 - \dfrac{1}{100})\ k\Omega$
- **Important**: The equivalent resistance of parallel resistors is always less than any of them.

Simplifying Resistor Networks

- Many resistor circuits can be simplified by alternately combining series and parallel resistors.

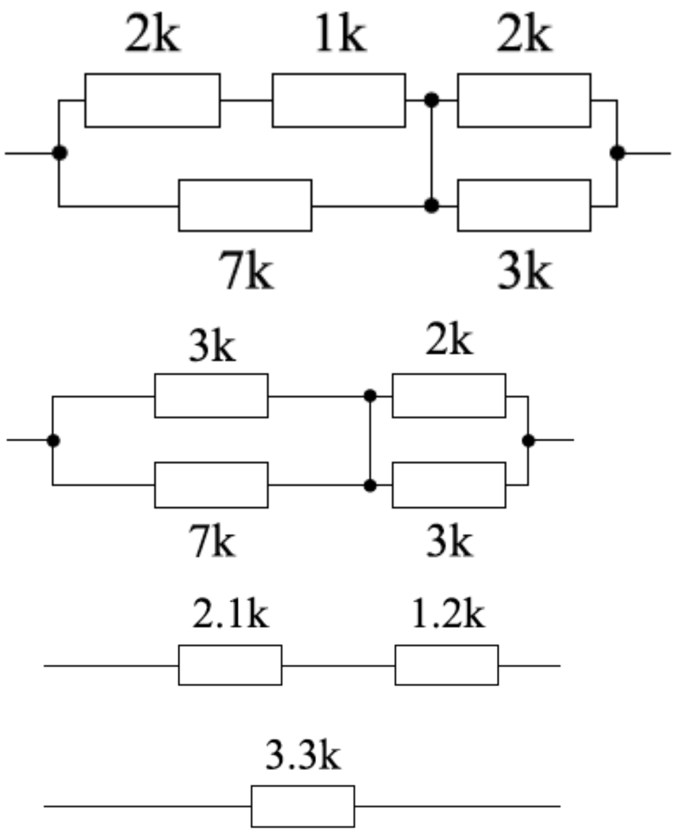
  Series: 2 kΩ + 1 kΩ = 3 kΩ

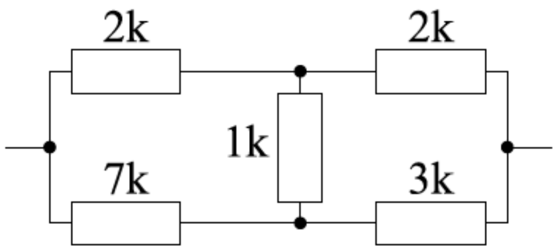
  Parallel: 3 kΩ || 7 kΩ = 2.1 kΩ

  Parallel: 2 kΩ || 3 kΩ = 1.2 kΩ

  Series: 2.1 kΩ + 1.2 kΩ = 3.3 kΩ

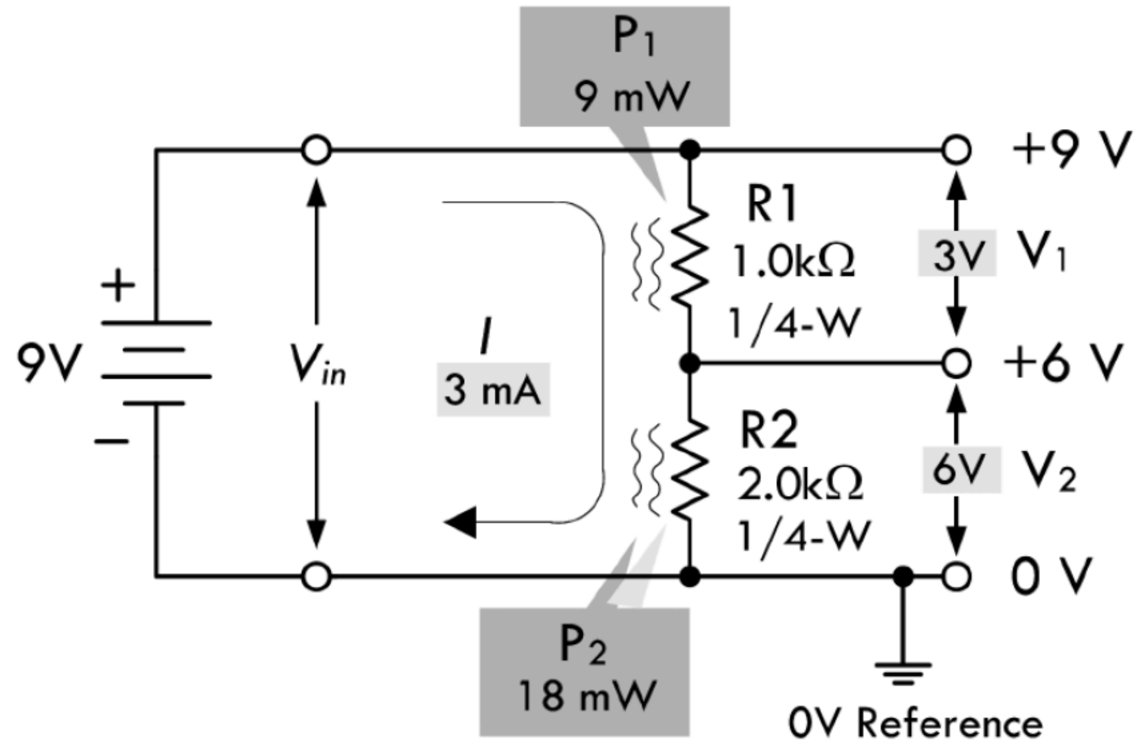
- Sadly this method does not always work: there are no series or parallel resistors here.





Example of a voltage divider

- Using two resistors R1 and R2, connected to a voltage source 9V, we can produce any voltage between 0V and the battery source voltage of 9V
- In this example, R1 and R2 are connected in series. The total resistance is 3kΩ
- The current I through the two resistors is therefore 9V/3k = 3mA.
- Therefore the voltage across R2 is: $V_2 = I \times R_2 = 6V$
- The voltage across R1 is 3V

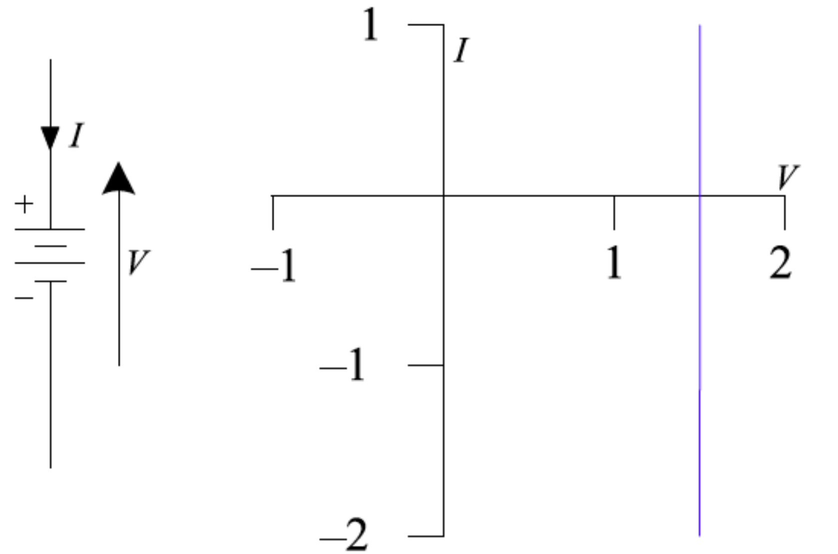


- This is called a voltage divider because R1 and R2 effectively divide the 9V into two parts!

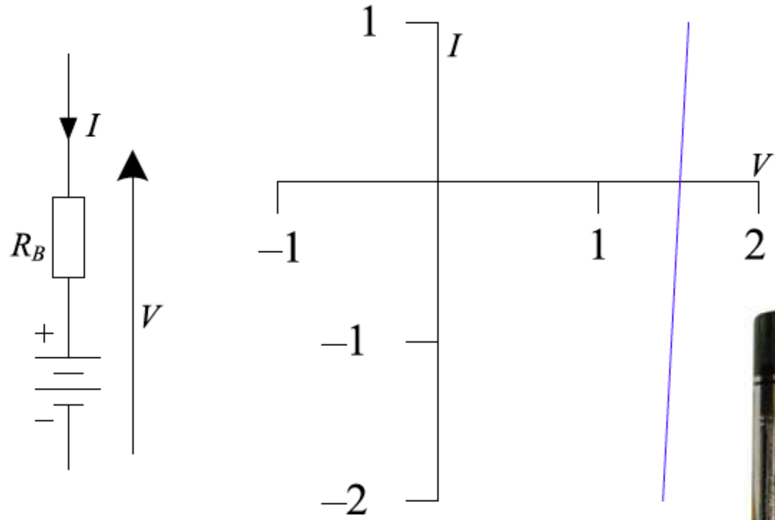
P55

Non-ideal Voltage Source

- An ideal battery has a characteristic that is vertical: battery voltage does not vary with current.
- Normally a battery is supplying energy so V and I have opposite signs, so I ≤ 0.



- An real battery has a characteristic that has a slight positive slope: battery voltage decreases as the (negative) current increases.
- Model this by including a small resistor in series. $V = V_B + IR_B$.



- The equivalent resistance for a battery increases at low temperatures.

Summary

- Identify resistor values
- Series and Parallel components
- Voltage and Current Dividers
- Simplifying Resistor Networks
- Battery Internal Resistance